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Note on the Application of Cascade Theory to Design of Axial-Flow Pumps

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Theoretical and experimental results are presented which reassure the usefulness of two-dimensional cascade theories to the design of axial flow pumps. For this purpose it is necessary to include the effect of the blade thickness upon the impeller flow which has been found to be responsible for reported discrepancies between predictions of thin airfoil theories and the performance of axial-flow pumps characterized by high stagger angle and low aspect ratio.

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APPROACH TO COMPRESSOR DESIGN

For a considerable time two-dimensional cascade theory, in conjunction with boundary-layer concepts, has been the basis of most axial-flow-fan and compressor-design approaches. These are extensively reviewed in a recent series of reports (1,2).[1] However, Bowerman, (3), has reported that this approach seems to be inadequate for certain types of axial-flow pumps. These pumps are characterized by low angle (or high stagger angle), relatively small ratio of blade height to axial extent of the chord, and solidities on the order of 0.75 to unity. The ratio of blade height to axial extent of the chord for the machine studied by Bowerman amounted to about one half, and he expected therefore considerable deviations from a two-dimensional cascade model. Bowerman employed a three-dimensional interference theory based upon the exact solution for a radial vortex line between a finite cylindrical hub and a case. The design was of the "free-vortex" type. Bowerman's experimental results as well as the cascade theories of Rannie (4) and Hlavka (5) are shown in Fig.1. It can be seen from this diagram that the test results compare favorably with the proposed three-dimensional theory but deviate 20-25 percent from the cascade theories.

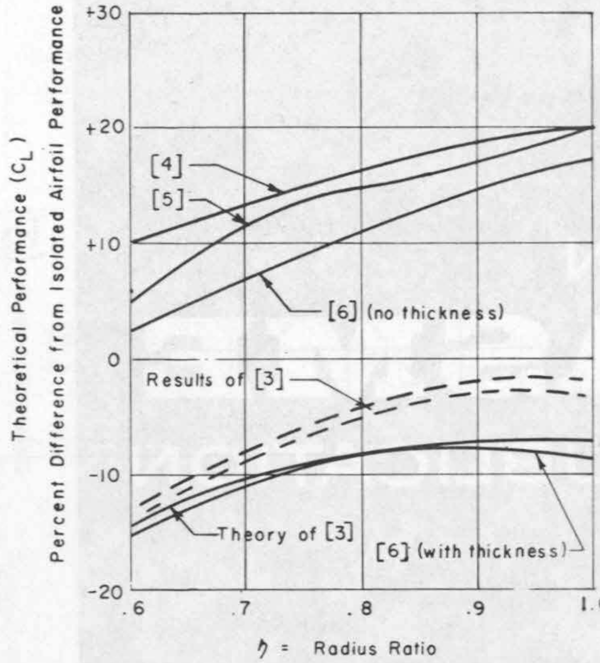

Fig.1 Comparison of theoretical expectations with and without thickness effect as percentage from isolated airfoil performance

The first is a linearized theory for flow through a cascade of circular arc airfoils of zero thickness. The second theory solves the same problem by more exact conformal mapping methods. Bowerman then concluded that two-dimensional cascade theories are not applicable to axial flow pumps of the type tested. He also mentioned that blade thickness might introduce some effect, but he believed it to be negligible for his investigation. (The blade thickness varied from 6 percent at the tip to 9 percent at the root.)

Since cascade theory has been extremely useful in the past, its abandonment in favor of a more complicated approach should be viewed with some reserve until the necessity of accurate performance calculations demands it. For this reason it was decided to investigate the performance of Bowerman's impeller in more detail and evaluate by application of more recent cascade theories (6) the effect of blade thickness and surface boundary layers on the lift coefficient and lift-slope curve.

From the survey paper of Schlichting (7) it can be seen that thickness is more important for pumps than typical compressor designs. In fact it was soon found in the present investigation that when the thickness is introduced into the cascade calculations of (6), the results of the theory agree closely with the experimental results of Bowerman as shown in Fig.1. Similar results were obtained with the charts of Mellor's work (8) but are not shown. Although these results confirm the validity of cascade theory at the one flow rate tested by Bowerman, it had been previously decided to re-evaluate the cascade performance of his impeller by means of detailed flow surveys. The objective of these measurements was to determine anew the lift coefficient and, in addition, the lift-slope relation and to compare these with the calculations previously mentioned. Surveys were, therefore, taken of the total pressure, flow angle, and static pressure downstream of the rotor and about one half chord downstream. Velocity profiles and streamline shifts were calculated from these data, and with this information the lift and drag coefficient for each section were determined at two, slightly different flow rates. In determining these quantities, an effective cascade plane was used at which one half of the measured streamline shift was assumed to have occurred.

RESULTS OF CALCULATIONS

The results of these calculations and the present measurements at the design flow rate (zero

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[1] Numbers in parentheses designate References at the end of the paper.

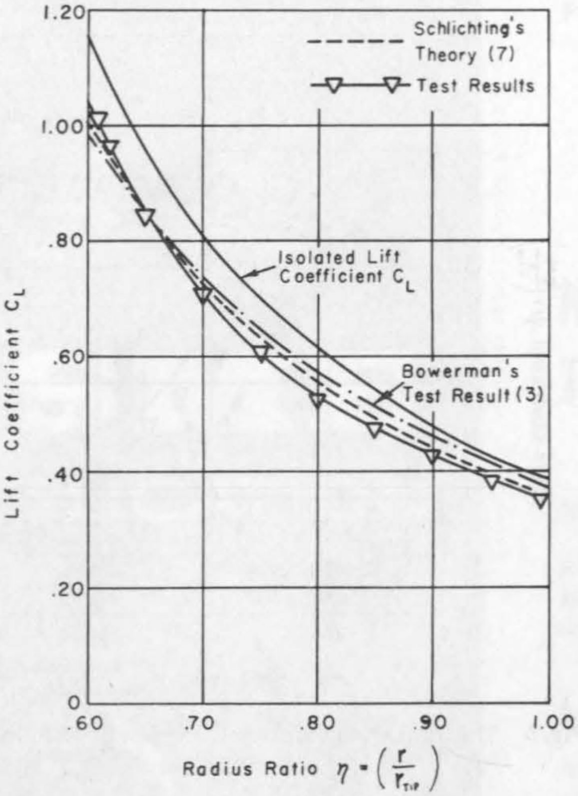

Fig. 2 Comparison of theoretical and measured lift coefficients

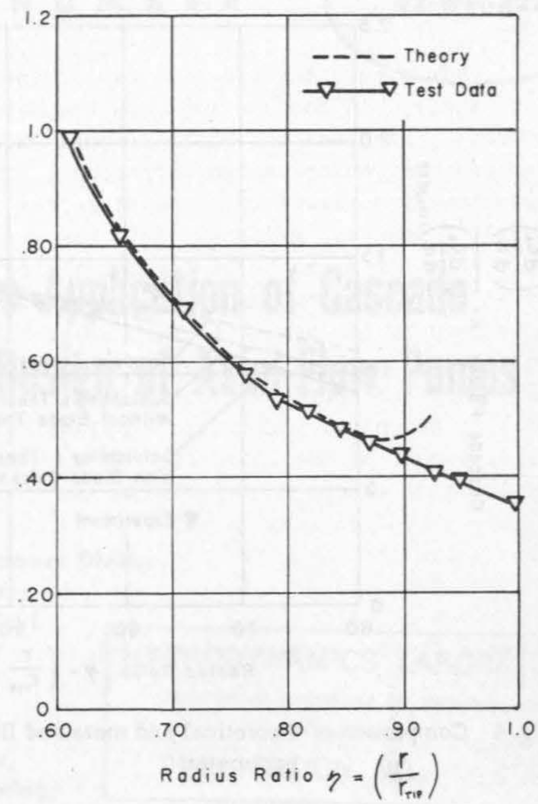

Fig. 3 Comparison of theoretical lift coefficients, corrected according to measured angles of attack, and measured lift coefficients

angle of attack) are shown in Fig.2. The measurements differ somewhat from those of (3), but are only slightly less than the theoretical calculations. Analysis of the flow surveys showed that the measured angle of attack was not zero for all stations of the impeller. The experimental values of lift coefficient were then corrected to zero angle of attack by Schlichting's theory and are shown in Fig.4. The agreement between the theory and experiment is now seen to be quite good except near the tip.

The drag coefficient deduced from the measurements was not too reliable, but in the vicinity of the mean radius it was found to be about 0.01.

The slope of the lift curve was determined from the survey measurements made at the two slightly different flow rates. The interval chosen was, in fact, womewhat small (about 3 percent of the flow rate), so that the resulting calculation of $dC_L/d\alpha$ did not have all the precision desired. Nevertheless, these results, shown in Fig.5, adequately demonstrate the validity of cascade theory for this pump geometry. For reasons not completely understood, the cascade factor k, the ratio of the lift slope in cascade to the isolated value, departs appreciably from the cascade theories shown in this figure in the outer and inner portions of the passage. The calculated drag coefficients in the same regions are also unusually large.

As a final comparison, the performance of the mean section ($r/r_2 = 0.825$) was calculated with cascade parameter calculated with Schlichting's theory for no friction and for an assumed drag coefficient of 0.01. These results, shown in Fig.6, again confirm in a reassuring way the use of cascade theory for the fairly extreme proportions of the present impeller.

The boundary layers on the surfaces of the blades may also be expected to have an influence on the lift coefficient in cascade. This point was investigated by taking into account the displacement effect of the surface boundary layers on the flow through a cascade of flat plates of the same solidity and stagger angle as the mean section of the present impeller. It was found that the displacement effect caused a shift of the zero lift angle of about 0.1 deg causing, thereby, a decrease in lift coefficient of only about 0.01 at the design point - a nearly negligible amount.

It is concluded that blade thickness is responsible for the discrepancy between predictions

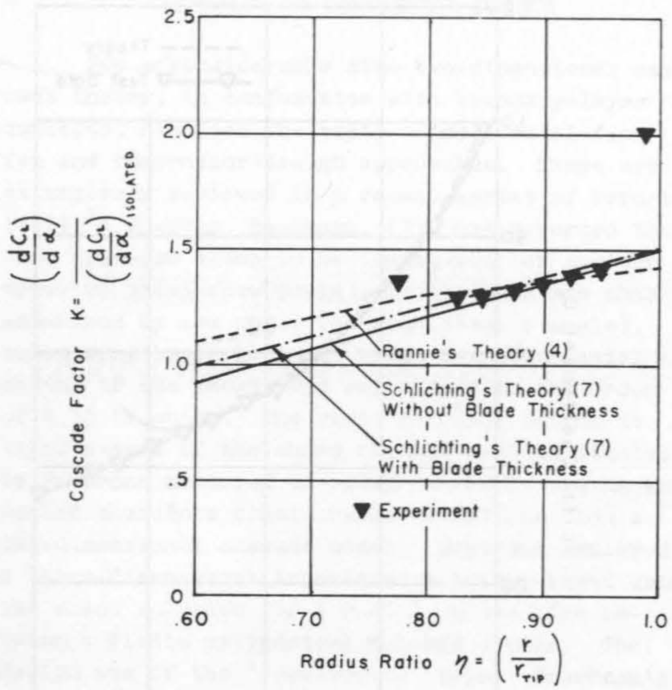

Fig. 4 Comparison of theoretical and measured lift-slope parameters

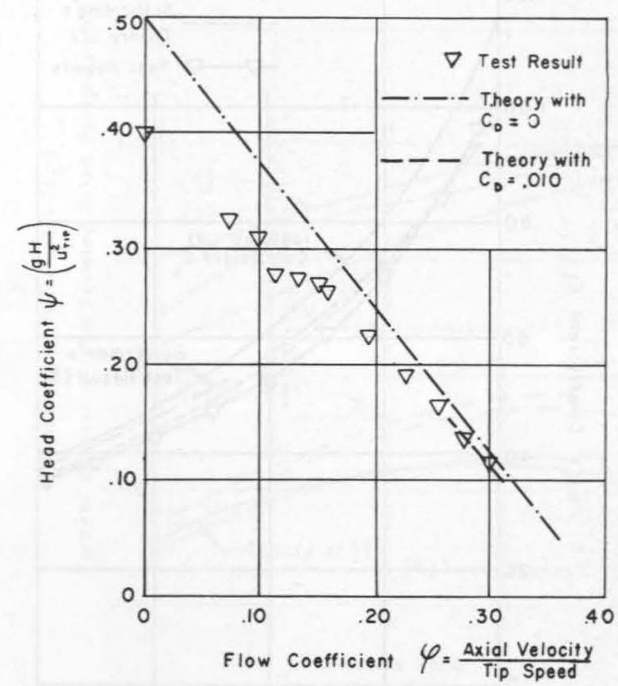

Fig. 5 Theoretical and measured off-design performance of mean streamline

of thin airfoil theories and the performance of axial-flow pumps of low aspect ratio. Not only were the measured lift coefficients in the pump predicted well by the theory of (6), but the slope of the lift curve agreed well with this theory where the measurements were reliable. The details of the present experimental work and analysis of the boundary-layer displacement effect in cascade will appear in a forthcoming publication.

ACKNOWLEDGMENT

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NOTATION

C_L = lift coefficient based on vector mean velocity

g = gravitational acceleration

H = head, ft

r = radius

u = peripheral speed

α = angle of attack measured from vector mean velocity